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E82-10204

CR-168631

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PR-1325-3

Quarterly Progress Report  
For the Period  
1 July to 30 September 1981

15 October 1981

(E82-10204) [BATHYMETRIC AND TECTONICS OF  
INDIAN OCEAN USING MAGSAT DATA] Quarterly  
Progress Report, 1 Jul. - 30 Sep. 1981  
(Analytic Sciences Corp.) 22 p  
HC A02/MF A01

N82-23577

Unclass  
CSC1 O&B G3/43 00204

Prepared Under:

Contract No. NAS5-26424  
(Magsat Scientific Investigation)

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
Goddard Space Flight Center  
Greenbelt, Maryland

RECEIVED

NOV 20, 1981

SIS/902.6

M-017  
TYPE II

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TABLE OF CONTENTS

	<u>Page No.</u>
1. INTRODUCTION	1-1
2. SUMMARY OF PROGRESS	2-1
3. TECHNICAL RESULTS	3-1
3.1 Data Quality and Preprocessing	3-1
3.2 Spectral Estimation Techniques and Coherence	3-5
3.3 Spectrum Modeling	3-10
REFERENCES	R-1

1.

INTRODUCTION

The TASC Magsat investigation covers an area in the eastern Indian Ocean containing several major bathymetric and tectonic features (Fig. 1.1-1). The overall objectives of this investigation are:

- Production of magnetic anomaly maps from Magsat data covering the study region (0 - 50 degrees S, 75 - 115 degrees E).
- Comparison of Magsat and satellite altimeter data (measuring geoid undulation) in this area, and quantification of their relationships.
- Determination of the optimum resolution of Magsat anomaly maps of the study region.
- Interpretation of Magsat data using satellite altimeter and other geophysical data in order to determine the origin and sources of the observed magnetic anomalies.

The investigation during this quarter focused on three areas: 1) an assessment of the data quality, especially characterization of spikes, 2) an analysis of the spectral passbands of significance to geophysical analysis, including computations of spectral coherence between nearby repeating Magsat tracks, and 3) a spectrum modeling effort designed to show the effect of spacecraft altitude on the observability of magnetic anomalies.

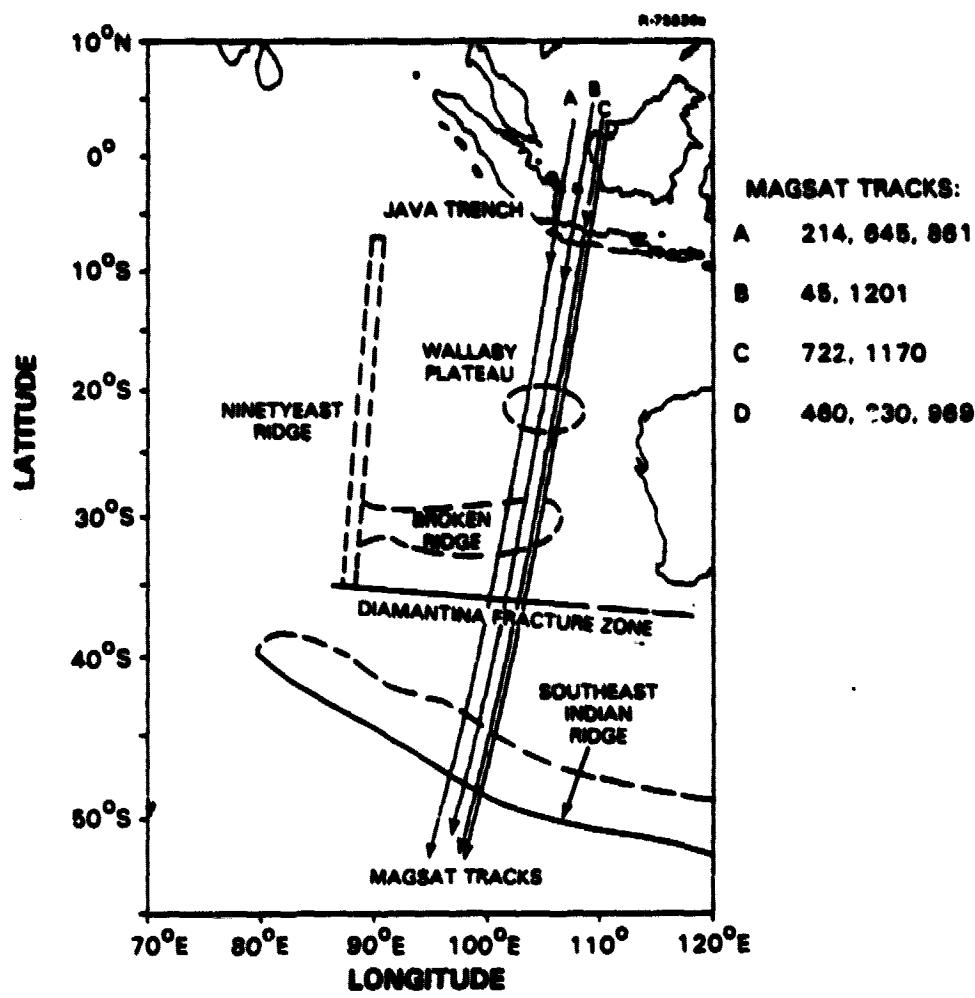


Figure 1.1-1 Study Area, With Selected Magsat Tracks

2.

SUMMARY OF PROGRESS

The work during this quarter emphasized analysis of the data quality and a further study of the spectral characteristics of the data. During the upcoming quarters, the data will be processed to generate an anomaly map of the project area using equivalent source dipoles. This anomaly map will then be compared to gravity data derived from satellite altimetry. The results of the present work will help in the design of data preprocessing and the choice of optimum grid spacing for anomaly map production.

• **CORRECTIONS FOR DATA GAPS (Section 3.1)**

Data gaps and spikes were identified and corrected. Bad data values occurring in the form of known missing data (flagged with a value of 99999.0) were corrected with a two-point interpolation. Smaller spikes evident to visual inspection are found to exist after removal of a field model.

• **SPECTRAL EFFECTS OF REMOVING FIELD MODEL, MEAN, AND RAMP (Section 3.1)**

The effect upon spectral estimates of removing a field model, mean (D.C. bias), and ramp (long wavelength components) was investigated numerically. The effects of these preprocessing schemes were analyzed for the impact to analysis of crustal anomaly fields.

• **COHERENCE BETWEEN ADJACENT TRACKS (Section 3.2)**

The repeatability of data along two satellite passes with closely-spaced ground tracks was studied to define the wavelength passband containing data of geophysical significance. The spectrum of the preprocessed data was then analyzed to determine the coherent bandwidth and noise floor. It was found that the spectrum of Magsat along-track data may be divided into 3 main passbands at wavelengths shorter

than those removed using the field model: 1) where the data from nearly repeating tracks have high coherence (i.e. a wavelength band where the data are repeatable and considered to be geophysically significant, 2800 km to 800 km), 2) where the data have low coherence but may have geophysical significance (800 km to 250 km), and 3) where the data has no geophysical significance with regard to crustal anomaly fields (< 250 km).

- **ALONG-TRACK SPECTRUM MODELING  
(Section 3.3)**

In order to understand the physical significance of along-track Magsat power spectra, a simple analytic model of these spectra was developed. In particular, a modified Bessel function model easily provides for upward continuation of along-track Magsat spectra to demonstrate their dependence on altitude.

These analyses yield a better understanding of the data and will result in a determination of optimum processing techniques to be used in the upcoming work. The remainder of this report will discuss the details of each of the above items. Also during the past quarter, work began on a manuscript for submission to Geophysical Research Letters. The results given in that manuscript will be more recent and may supersede some results presented here.

3.

TECHNICAL RESULTS

3.1 DATA QUALITY AND PREPROCESSING

Before large scale processing of Magsat data can be accomplished, it is necessary to evaluate the data quality and determine what preprocessing, if any, is required. Optimum computational methods and parameters (e.g. the minimum spacing for equivalent source dipoles) can then be determined. This section will discuss the effects of data preprocessing on spectrum results.

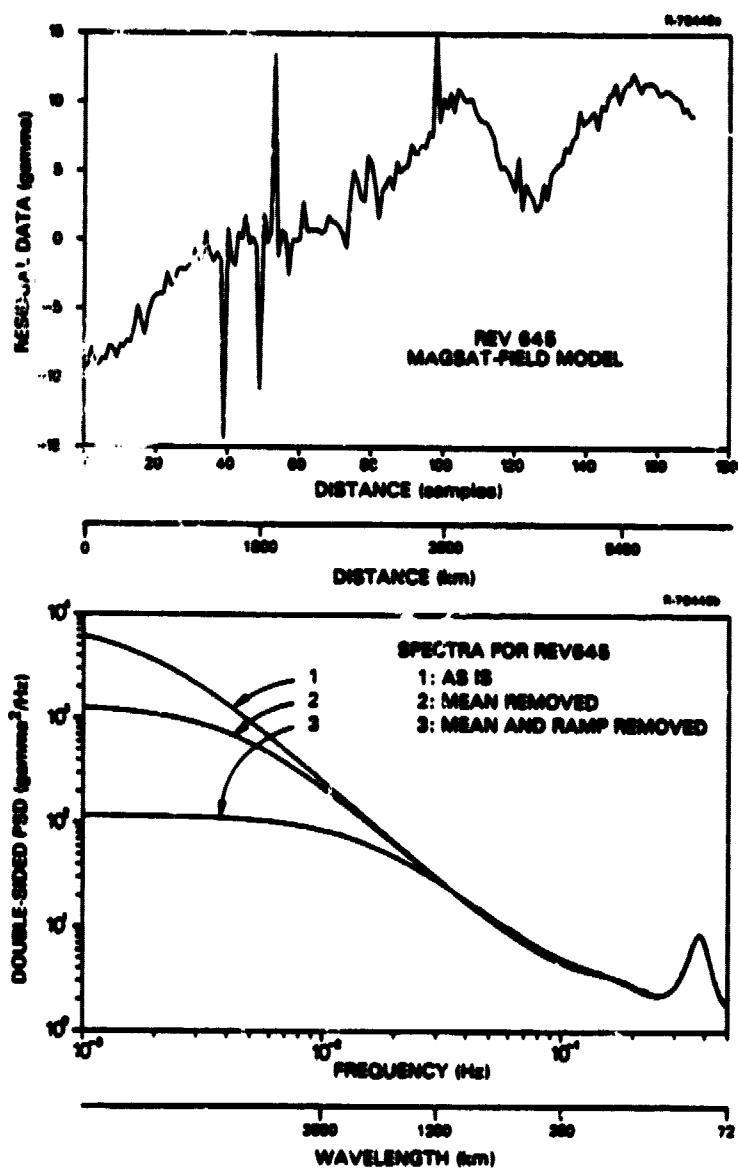
Twenty-four revs of 80 point averages of scalar magnitude data crossing the project area (Fig. 1-1.1) were analyzed. All revs had 180 points or fewer, and none had fewer than five missing points (flagged with 99999.0). In order to maintain a uniformly spaced data series, the actual value of these points must be estimated and inserted into the time series. For the present, two-point linear interpolation was used to estimate the missing data values. The time series were then processed to remove a 14th order field model (MGST4/81), and spikes of up to 15 gammas are found to contaminate the residual data. It is not likely these smaller spikes are real, as their spatial wavelengths are shorter than the resolution limit for the data (Ref. 1). Since a universally effective algorithm for the identification, discrimination and replacement of single-point spikes has not yet been developed, these were left in place for the time being. Numerical tests have shown that their presence does not significantly change the conclusions presented here.

Effective analysis of crustal anomaly fields requires a knowledge of both the wavelength passband which contains the desired geophysical information and the effective passband where significant information exists in the data. Knowledge of these passbands will permit determination of the optimum spacing for the grid of equivalent dipole sources. As a first step in defining these passbands, the effects of ramp removal on spectral estimates will be shown.

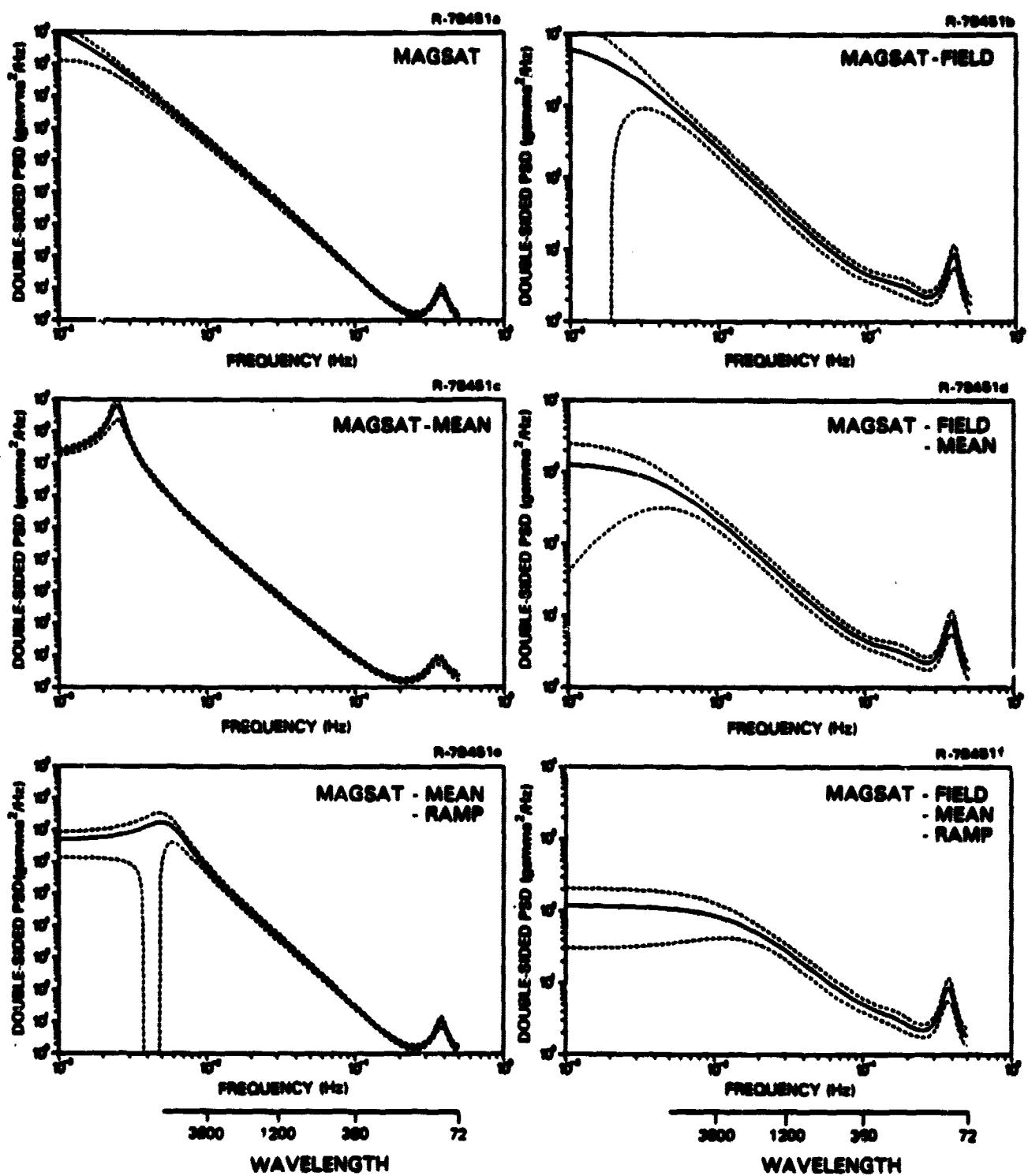
The common practice of preprocessing Magsat anomaly data by removing a mean and ramp alters the spectral content of the data. Subtracting a field model of degree and order 14 removes wavelengths longer than about 2800 kilometers, primarily attributable to the core field. Since this procedure removes the core field, the residuals are crustal magnetic anomalies plus external field effects. The external field effects are difficult to model, but are at longer wavelengths than crustal anomalies. External field effects can sometimes be removed by subtracting a ramp from a pass of the residual data. The effect of removing a mean or mean and ramp is illustrated in Figure 3.1-1.

Further illustration of mean and ramp removal is shown in Figure 3.1-2. The slope in the midsection of the spectra, and the noise floor are unchanged because mean and ramp removal are a form of high-pass filtering.

Subtracting the mean from the data removes any bias which may be due to instrument error or a uniform external field. Subtracting a ramp from the data is expected to remove primarily the effects of external current fields. However, ramp removal will also take out any long-wavelength regional components that may exist in the crustal field.



**Figure 3.1-1** Rev 645 showing: a) The scalar magnitude data minus the field model. b) The autoregressive spectral estimate of order 5. The spectra for the above data (curve 1), mean removed (curve 2), and mean and ramp removed (curve 3). Note there is no significant difference among the 3 spectral estimates at frequencies greater than about 0.03 Hz (1050 km). Considering the confidence bounds of these different estimates, they are in agreement for wavelengths longer than about 1900 km.



**Figure 3.1-2** Rev 645 is here shown with the Magsat data on left and Magsat minus field model on right. (Note the different vertical scales.) Top to bottom rows are: unaltered data, mean removed, and mean and ramp removed. The dotted lines bound the 95% confidence level.

The conclusion is that, for the tracks shown in Fig. 1.1-1, the wavelength passband from 2800 km to 1050 km (extending to shorter wavelengths than contained in the field model) is modified by the removal of a mean or a mean and a ramp. The noise floor appears to be reached at wavelengths slightly shorter than 250 km. Thus, the region from 250 km to 1050 km contains information which may pertain to the geophysics of the region and is unaltered by the removal of a mean or a mean and a ramp. The tradeoff invoked by mean or ramp removal involves decreasing or eliminating the effects of external fields at the expense of perturbing the data with wavelengths longer than about 1050 km in the TASC investigation area.

### 3.2 SPECTRAL ESTIMATION TECHNIQUES AND COHERENCE

Magsat tracks which are geographically separated by less than about 100 km along the entire track segment are considered repeat tracks. Some of these are shown in the map of Fig. 1.1-1. This separation, and the altitude variation, are both small compared to the geophysically significant correlation distances in major crustal magnetic anomalies. Within the usable wavelength passband (1050 to 250 km) described in Section 3.1, computations were performed to find where the data are repeatable from track to track (high coherence), and where they are not (low coherence).

In this section, the concept of repeatability is quantified and illustrated in the frequency domain by computations of spectral coherence between nearly repeating tracks. These computations have defined two bands of differing behavior within the usable data passband (1050 to 250 km). A high coherence passband (1050 to 800 km) defines the region where the data are repeatable and the spectrum of the data can be considered to contain geophysically

significant information. A low coherence passband (800 to 250 km) defines that part of the data which is not repeatable from track to track, but which may still contain useful data along single tracks. The low coherence in this passband is partly due to track separation and may also be due to colored ( $f^{-n}$ ) noise.

A quantitative measure of repeatability is provided by the spectral coherence function (Ref. 2) which is the squared correlation coefficient of the data along the tracks at each frequency (or wavelength) in the spectrum. Spectral coherence,  $\rho(\omega)$ , is defined as follows:

$$\rho(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega)S_{yy}(\omega)} \quad (3.2-1)$$

where, for the two tracks  $x(t)$  and  $y(t)$ ,  $S_{xx}$  and  $S_{yy}$  are the corresponding auto-spectra and  $S_{xy}$  is the cross-spectrum. Autoregressive methods of spectrum estimation are discussed in Ref. 3.

The coherence between pairs of repeat tracks is reduced from the maximum value of 1 by non-repeating errors or noise in the individual measurements. The observed Magsat data is composed of the signal (considered here to be crustal anomalies) and noise (e.g. instrument noise, orbit errors, time-varying external fields). The noise is the major contributor to the non-repeatable portions of the spectra.

An autoregressive spectral estimation algorithm was applied to the sections of Rev 645 (previously described) and Rev 861. These two tracks are separated by 49 to 92 km in surface distance, and by 70 to 80 km in altitude. This

spectral estimation algorithm differs from that used in Section 3.1 in that it uses information from both tracks to estimate the spectra of either track. Thus, where the two tracks are coherent, the two-channel spectral estimate is better (as it operates on more data). This algorithm then estimates the coherence between the two tracks (eq. 3.2-1), which is a measure of how well one track can be predicted from the other. The result is shown in Figure 3.2-1. The anomaly profiles are visually comparable, and they both have a few spikes (occurring at different sample points). Removal of the spikes causes the estimated spectra to agree more closely, but does not significantly change the conclusions that may be drawn from the coherence.

The spectral structure in Fig. 3.2-1 is more detailed than shown and discussed in Section 3.1 mainly because the spectral estimation algorithm chose order 10 as the best fit here, versus order 5 chosen previously. This may be partially due to the increase of the available data (two tracks). A spectral peak at about 1700 km is clearly evident here, and was not seen with the order 5 model. The cause of this peak is a nearly periodic structure of the data, as may be seen in the time series of Fig. 3.2-1.

Choosing a squared coherence level at 0.5 as an arbitrary cutoff between high coherence and low coherence data, the short wavelength limit for coherent data occurs at about 800km. The long wavelength limit for coherent data (the maximum coherence occurs at about 1600 km) is not considered significant, since unlike the short wavelength limit, it is very different among different pairs of repeat tracks, and it falls within a spectral band affected by removal of a field model, mean and ramp.

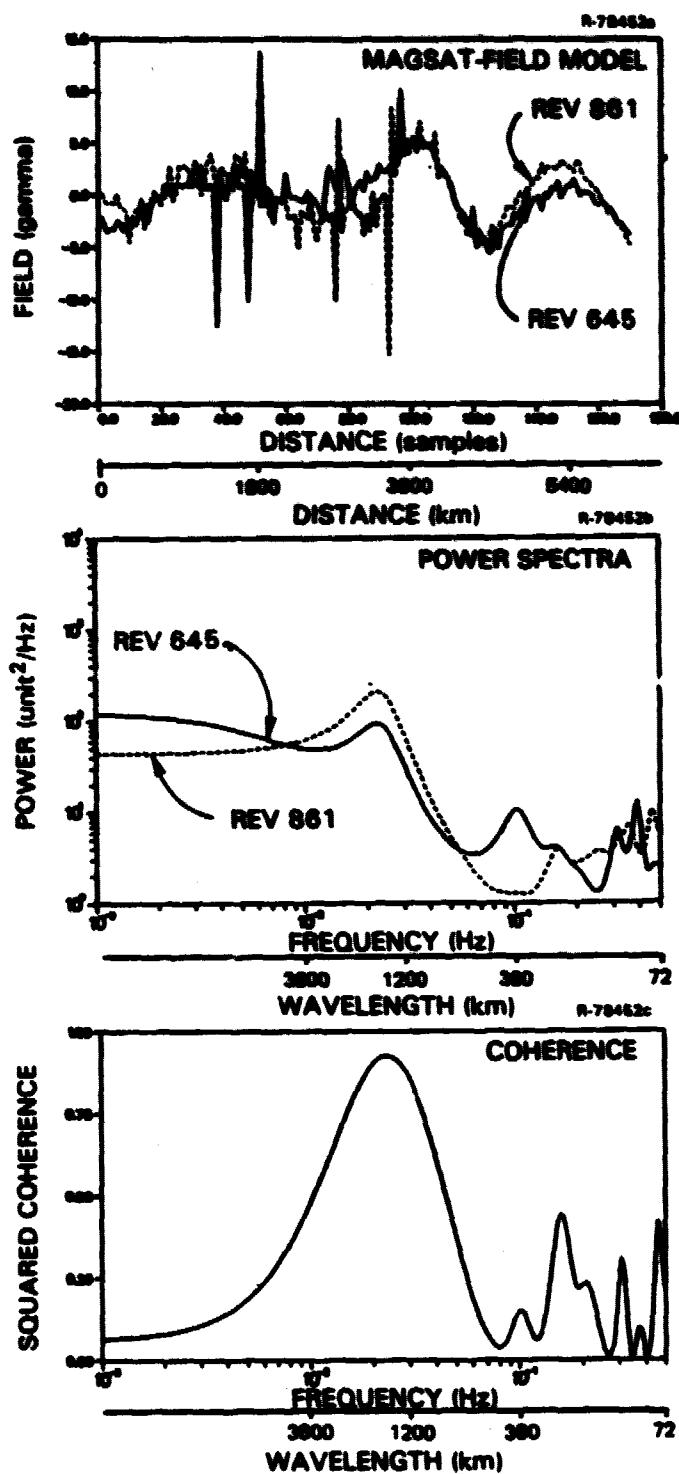


Figure 3.2-1

Top: two repeat tracks, Rev 645 (solid line) and Rev 861 (dashed line), linear trends removed.  
 Middle: the estimated spectra, where the two-channel auto-regressive spectral estimation algorithm chose order 10 for the best fit to the data.  
 Bottom: Spectral coherence between the two tracks.

Work is in progress to add confidence bounds to the coherence estimates. In the future, the confidence bounds will be used to define the wavelength band in which there is significant coherence.

The noise floor in these spectra is unchanged from that observed in Section 3.1. The data used for these analyses were taken during magnetically quiet times. If the magnetic environment varied significantly between the repeat tracks, the added noise would be incoherent, thus degrading the overall coherence.

The conclusion from this section is that wavelengths longer than about 800 km are highly coherent, while shorter ones are not. Figure 3.2-2 summarizes these observations about the usable information content of each wavelength band.

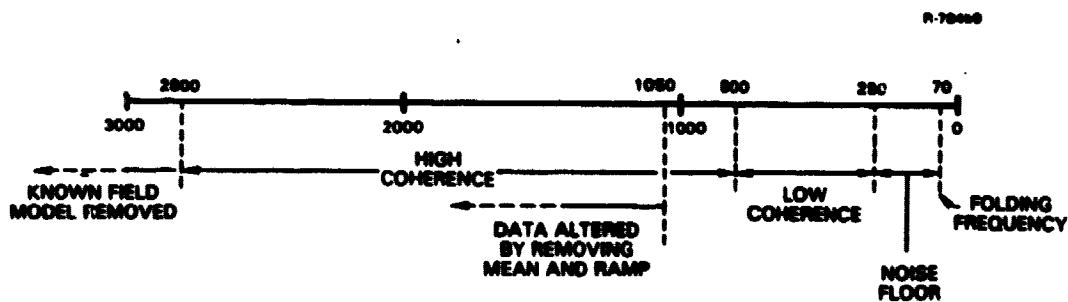


Figure 3.2-2 Properties of different regions of the Magsat along-track power spectrum, as determined from spectrum analysis and coherence studies.

The low coherence in the band between 800 and 250 km does not mean that geophysical information cannot be obtained in this band. This could be due to variations in altitude and geographic position of the repeat tracks. The properties of

this band must be investigated further with gridded data and by comparison with gravity data.

The results of this and the previous section suggest that low-pass filtering and decimation of Magsat data into sampling intervals substantially longer than the 80 sample averages now available on the Investigator tapes would not degrade the analysis of crustal magnetic anomalies. Low-pass filtering and decimation over as many as 500 to 600 samples could result in more cost-effective data analysis as it would only delete incoherent noise.

### 3.3 SPECTRUM MODELING

The along-track power spectrum of Magsat data has been estimated and used as a tool to study the data quality and resolution capability. It would also be useful, for interpretation purposes, to know what these spectra are expected to look like for reasonable physical models of the earth's magnetic field. Analytic models for these spectra could allow study of the effects of instrument noise, external fields, and change in altitude. In this section, some preliminary results of model development are described. The purpose of this work was to develop a simple analytic model of along-track spectra that could both describe the data and be easily upward continued, in order to demonstrate the magnitude of the effects of satellite altitude on the power spectrum. This is important because features in the spectrum representing crustal magnetic anomalies are attenuated rapidly with altitude and are obscured by noise when their power drops below a certain level. Further work is necessary to relate the parameters of this initial model to physical quantities.

Upward continuation of a spherical harmonic power spectrum (the  $R_n$  defined by Lowes, 1974, Ref. 4) is accomplished by multiplication of the individual terms of degree  $n$  by the factor  $(a/r)^{(2n+4)}$  (where  $a$  is the earth's radius and  $r$  is the radius of the observation point). For small values of altitude and for high degree, this spherical harmonic power spectrum,

$$R_n = (n+1) \sum_{m=0}^n [(g_n^m)^2 + (h_n^m)^2] \quad (3.3-1)$$

asymptotically approaches a continuous flat-earth spectrum. For cases where the flat-earth approximation is valid ( $r-a \ll a$ ,  $n \gg 1$ ), a two-dimensional power spectrum,  $\Phi(\omega_x, \omega_y)$ , is upward continued by multiplying by the factor  $\exp[-h(\omega_x^2 + \omega_y^2)^{1/2}]$ . Thus,

$$\Phi(\omega_x, \omega_y, r) = \Phi(\omega_x, \omega_y) \cdot \exp(-2h\Omega) \quad (3.3-2)$$

where  $h=r-a$  is the height, and  $\Omega^2 = \omega_x^2 + \omega_y^2$ .

For along-track, or great-circle spectra, (such as would be observed along tracks of aeromagnetic or satellite data), upward continuation is not so straightforward. Formulas relating great-circle spectra to spherical harmonic spectra, and along-track spectra to two-dimensional isotropic spectra, have recently become available but have not been widely used. Thus, for example, the along-track (great-circle) aeromagnetic spectrum of Alldredge et al. (Ref. 5) was not properly interpreted in many of the older papers which tried to relate these observations to global magnetic models. Recently, McLeod and Coleman (1980, Ref. 6) derived expressions for the relationships between great-circle and spherical harmonic spectra and applied these to model satellite observations.

To derive a simple possible model of Magsat along-track power spectra which permits upward continuation, the following assumptions are made: 1) the flat-earth approximation is valid for features in the wavelength range of interest, 2) the two-dimensional power spectrum is isotropic and has the form of an exponential. These assumptions allow the development of an analytic expression to describe the along-track power spectrum at any altitude. The second assumption is equivalent to stating that the two-dimensional power spectrum at any altitude may be described by the following function:

$$\Phi(\Omega) = c \exp(-b\Omega) \cdot \exp(-2\Omega h) \quad (3.3-3)$$

where  $c$  and  $b$  are parameters,  $h$  is altitude, and  $\Omega$  is radial frequency for the two-dimensional isotropic spectrum. This model is an "attenuated white noise" (AWN) type model since  $\Phi(\Omega)=c$  at a depth of  $h=-b/2$  (Ref. 7).

It can be shown that the along-track power spectrum,  $S(\omega)$ , corresponding to this two-dimensional power spectrum is the Abel Transform of Eq. 3.3-3 (Ref. 8). Thus,

$$S(\omega) = \frac{1}{\pi} \int_{\omega}^{\infty} \frac{\Omega \Phi(\Omega)}{\sqrt{\Omega^2 - \omega^2}} d\Omega \quad (3.3-4)$$

The solution of this integral for the AWN model is:

$$S(\omega) = \frac{1}{\pi} \omega K_1[\omega(b+2h)] \quad (3.3-5)$$

where  $K_1$  is the modified Bessel function (Ref. 9).

A choice of values for the parameters  $b$  and  $c$  was made by fitting equation 3.3-5 to an observed Magsat power

spectrum at  $h = 300$  km. (The modeling discussed here is being done for the spectrum of  $|B|$ .) The values obtained were  $c = 1.0 \times 10^8$  and  $b = 6.37 \times 10^{-1}$ . With these parameter values for  $b$  and  $c$ , the function of eq. 3.3-5 is plotted in Fig. 3.3-1 for altitude values of  $h=150$ , 300, and 450 km. If it is assumed that the noise floor, representing white noise, is at the level of approximately  $0.2 (\gamma)^2/(\text{cy/sample})$ , then the resolution limit of satellite data would be approximately 160 km for  $h = 150$ , 350 km for  $h = 300$  km, and 540 km for  $h = 500$  km.

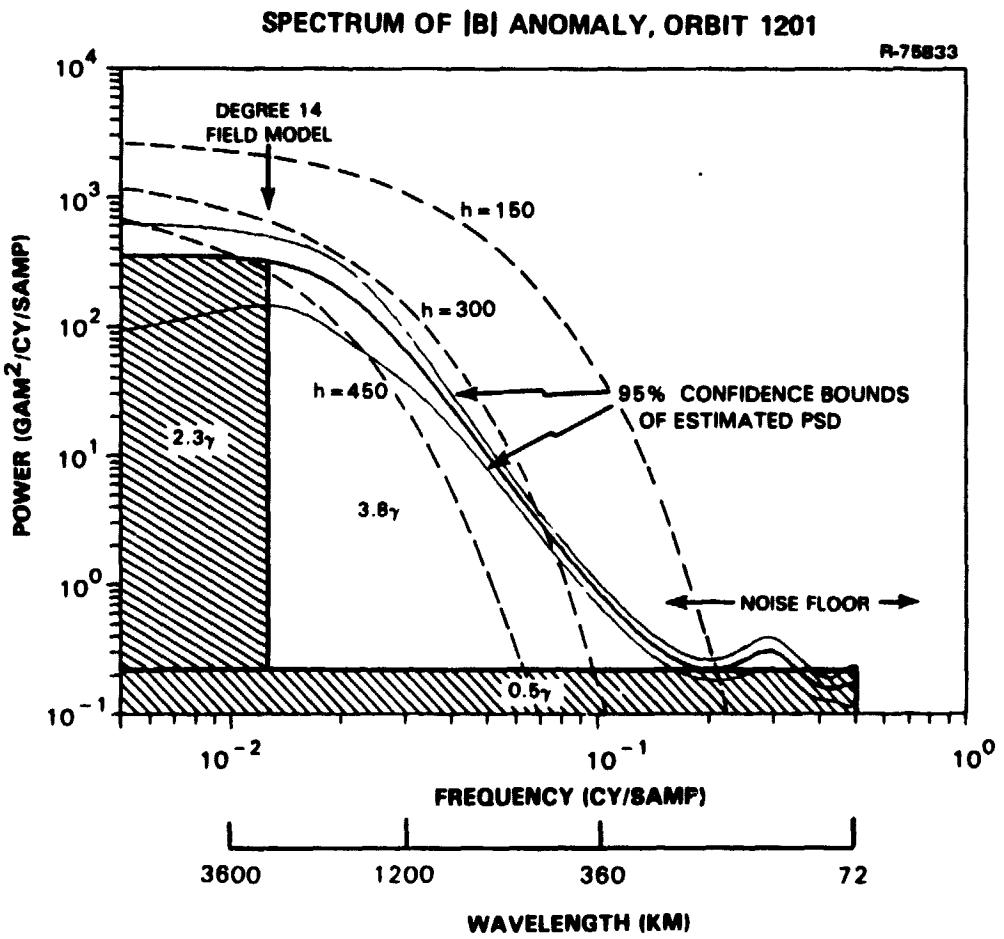


Figure 3.3-1 Observed Magsat spectrum for  $|B|$  of orbit 1201 compared with values given by the analytic model of Eq. 3.3-5. The effect of different heights  $h$ , is shown. (Orbit 1201 was at  $h=350$  to 360 km during the pass over the investigation region.)

This model has certain simplifications: most notably the flat-earth approximation and isotropy of the two-dimensional power spectrum. However, the estimates of the resolution capability at different altitudes seem reasonable. The formula of eq. 3.3-4 is very similar to the formula of eq. 27 of McLeod and Coleman (1980, Ref. 6), which is the asymptotic approximation of an expression describing the relationship between an average great-circle power spectrum and the spherical harmonic spectrum. Further work will give the asymptotic form of the spherical harmonic spectrum,  $R_n$ , which is implied by the flat-earth model of eq. 3.3-5.

When plotted on a log-linear scale, the function (eq. 3.3-5) has the form of a straight line with a slope=-2h, indicating that it is essentially exponential. This is the form of spectra observed in aeromagnetic surveys as modeled by Spector and Grant (1970, Ref. 10) for the purpose of determining depth to magnetic basement. The fact that our along-track spectrum estimates from Magsat data show power law, rather than exponential behavior, needs further investigation. This difference is important because many models have assumed the exponential form. Reasons for this disagreement could be: 1) the crustal field is not exponential and 2) time-varying noise sources.

#### PLANS FOR THE NEXT REPORTING PERIOD

The Magsat data will be prepared for computing crustal anomaly maps by modeling the equivalent dipoles in the project area. Special emphasis will be placed on the removal of spikes and proper filtering and preprocessing of the data. Since two more investigator tapes have been recently received (Sept. 30), production processing can begin on the complete data set.

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The first runs of the equivalent source fitting program will be based on a 7 degree spacing (as data is highly coherent to a short wavelength limit of 800 km - Section 3.3).

COST DATA

Total expenditures through 30 September have been \$23,426. Expenditures during the quarter 1 July through 30 September were \$12,093.

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